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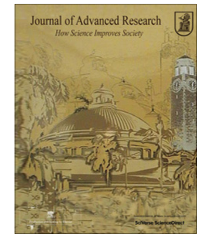

ORIGINAL ARTICLE

Non-uniform cosine modulated filter banks using meta-heuristic algorithms in CSD space

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A R T I C L E I N F O

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A B S T R A C T

This paper presents an efficient design of non-uniform cosine modulated filter banks (CMFB) using canonic signed digit (CSD) coefficients. CMFB has got an easy and efficient design approach. Non-uniform decomposition can be easily obtained by merging the appropriate filters of a uniform filter bank. Only the prototype filter needs to be designed and optimized. In this paper, the prototype filter is designed using window method, weighted Chebyshev approximation and weighted constrained least square approximation. The coefficients are quantized into CSD, using a look-up-table. The finite precision CSD rounding, deteriorates the filter bank performances. The performances of the filter bank are improved using suitably modified meta-heuristic algorithms. The different meta-heuristic algorithms which are modified and used in this paper are Artificial Bee Colony algorithm, Gravitational Search algorithm, Harmony Search algorithm and Genetic algorithm and they result in filter banks with less implementation complexity, power consumption and area requirements when compared with those of the conventional continuous coefficient non-uniform CMFB.

Introduction

Filter banks are extensively used in different applications such as compression of speech, image, video and audio data, trans-multiplexers, multi carrier modulators, adaptive and bio signal processing [1]. Filter banks decompose the spectrum of a given signal into different subbands and each subband is associated with a specific frequency interval. In certain applications such as wireless communications and subband adaptive filtering, a non-uniform decomposition of subbands is preferred [2–5].

Design of filter banks with good frequency response characteristics and reduced implementation complexity is highly desired in different applications. Multipliers are the most expensive components for implementing the digital filter in hardware. The multipliers in the filters can be implemented using shifters and adders, if the coefficients are represented by signed power of two (SPT) terms [6]. Canonic signed digit (CSD) representation is a special case of SPT representation [7]. It contains minimum number of SPT terms and the adjacent digits will never be both non-zeros. As a result, efficient implementation of multipliers using shifters/adders is possible [7].

Different methods exist for the design of non-uniform filter banks (NUFB). In one approach, two channel filter banks are used as building blocks and a tree structured filter bank is generated for getting non-uniform band splitting [1]. In the second approach, one or more prototype filters are designed and all the other filters are obtained by cosine or DFT modulation [8–10]. In another approach, called recombination technique, the analysis filters of an M channel uniform filter bank are

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combined with the synthesis filters of a different filter bank having smaller number of channels [11].

A simple and efficient design of NUFB is by the cosine modulation of the prototype filter and combining appropriate filters of the resulting uniform filter bank [10]. The non-uniform CMFB design is derived from a uniform CMFB. Hence the attractive properties of a uniform CMFB are retained in the non-uniform CMFB. Only the prototype filter need to be designed and optimized. All the other analysis and synthesis filters with unequal bandwidths are obtained from this filter, by merging appropriate filters of the uniform filter bank. The prototype filter is designed using non-linear optimization in [10]. A modified approach, in which the prototype filter is designed using linear search technique was given in Zijing and Yun [12].

Cosine modulated filter banks (CMFB) are one popular class among the different M-channel maximally decimated filter banks [13–15]. In perfect reconstruction (PR) filter banks, the output will be a weighted delayed replica of the input. In case of near perfect reconstruction (NPR) filter banks, a tolerable amount of aliasing and amplitude distortion errors are permitted. Design of NPR CMFB is easier and less time consuming compared to the corresponding PR CMFB. Even though small amounts of aliasing and amplitude distortion errors exist, these filter banks are widely used in different applications due to the design ease [16–19]. It is difficult to attain high stopband attenuation with PR CMFB. Hence as a compromise, NPR structures can be preferred in those applications, where some aliasing can be tolerated.

In multiplier-less filter banks, the filter coefficients are represented by signed power of two terms (SPT) and the multiplications can be carried out as additions, subtractions and shifting. Canonic signed digit (CSD) representation is a special form of SPT representations and is a minimal one. But CSD representation of the coefficients may lead to deterioration of the filter performances. Hence suitable optimization techniques have to be deployed to improve the performances. Multiplier-less design of NPR non-uniform CMFB with conventional FIR filter as the prototype filter and the coefficients synthesized in the CSD form using modified meta-heuristic algorithms is hitherto not reported in the literature.

In this paper a new approach for the design of multiplier-less NPR non-uniform CMFB is given, in which the prototype filter is designed using different techniques such as window method, weighted Chebyshev approximation and weighted constrained least square method. The coefficients are quantized using canonic signed digit (CSD) representation. The CSD rounding deteriorates the filter bank performances. The finite precision performances of the filter bank in the CSD space can be made at par with those of infinite precision, using various modified meta-heuristic algorithms. To improve the frequency response characteristics of the filters, optimization in the discrete domain is required. Conventional gradient based approaches cannot be deployed here, as the search space is discrete. Meta-heuristic algorithm is a proper choice for such problems [20] to result in global solutions by properly tuning the parameters.

The remaining part of the paper is organized as follows: Section 'Cosine modulated uniform filter banks' gives an introduction of NPR CMFB. Section 'Cosine modulated non-uniform filter banks' briefly illustrates the design of non-uniform NPR CMFB. Section 'Design of prototype filter' gives a brief description of the different prototype filter designs for the NPR CMFB. Section 'Multiplier-less design of non-uniform CMFB' explains the design of CSD coefficient CMFB. Section 'Optimization of non-uniform CMFB using modified meta-heuristic algorithms' outlines the optimization of the CSD coefficient filter bank using various modified meta-heuristic algorithms. Result analysis is given in Section 'Results and discussion' and the conclusion in Section 'Conclusion'.

Cosine modulated uniform filter banks

In an M-channel maximally decimated uniform CMFB, the input signal is decomposed into subband signals having equal bandwidths. A set of M analysis filters $H_k(z),\ 0 \leqslant k \leqslant M-1$ decomposes the input signal into M subbands, which are in turn decimated by M fold downsamplers. A set of synthesis filters $F_k(z),\ 0 \leqslant k \leqslant M-1$ combines the M subband signals after interpolation by a factor of M on each channel. The reconstructed output, $Y(z)$ is given by Eq. (1) [1].

$$Y(z) = T_0(z)X(z) + \sum_{l=1}^{M-1} T_l(z)X(ze^{-j2\pi l/M}) \tag{1}$$

where $T_0(z)$ is the distortion transfer function and $T_l(z)$ is the aliasing transfer function.

$$T_0(z) = \frac{1}{M}\sum_{k=0}^{M-1} F_k(z)H_k(z) \tag{2}$$

$$T_l(z) = \frac{1}{M}\sum_{k=0}^{M-1} F_k(z)H_k(ze^{-j2\pi l/M}) \tag{3}$$

$$l = 1, 2, \ldots, M-1$$

The analysis and synthesis filter responses are normalized to unity. Hence as given in Koilpillai and Vaidyanathan [21]

$$(1-\delta_1) \leqslant |MT_0(e^{j\omega})| \leqslant (1+\delta_2) \tag{4}$$

Amplitude distortion error is given by

$$E_r = \max_{\omega} |[|MT_0(e^{j\omega})| - 1]| \tag{5}$$

The worst case aliasing distortion is given by

$$E_a = \max_{\omega}(T_{alias}(\omega)) \tag{6}$$

where

$$T_{alias}(\omega) = \left[\sum_{l=1}^{M-1} |T_l(e^{j\omega})|^2\right]^{\frac{1}{2}} \tag{7}$$

For the design of NPR CMFB, a linear phase FIR filter with good stopband attenuation and which provides flat amplitude distortion function is initially designed. All the analysis and synthesis filters are generated from this prototype filter by cosine modulation. All the coefficients are real. The coefficients of the analysis and synthesis filters are given by Eqs. (8) and (9) respectively [1].

$$h_k(n) = 2p_0(n)\cos\left(\frac{\pi}{M}(k+0.5)\left(n-\frac{N}{2}\right) + (-1)^k\frac{\pi}{4}\right) \tag{8}$$

$$f_k(n) = 2p_0(n)\cos\left(\frac{\pi}{M}(k+0.5)\left(n-\frac{N}{2}\right) - (-1)^k\frac{\pi}{4}\right) \tag{9}$$

$$k = 0, 1, 2, \ldots, M-1$$

$$n = 0, 1, 2, \ldots, N-1$$

Different techniques are available for the design of the optimal prototype filter of the NPR CMFB using different objective functions and using different FIR filter approximations. Since the prototype filter is cosine modulated to obtain the analysis and synthesis filters, the filter bank design is reduced to the optimal design of the prototype filter. If the prototype filter has linear phase response, then the overall filter bank will have linear phase response. The adjacent channel aliasing cancelation is inherent in the filter bank design. Remaining is the aliasing between non-adjacent channels. Prototype filter with good stopband attenuation reduces the aliasing between the non-adjacent channels. The 3-dB cut-off frequency of the prototype filter should be at $\omega_{c,3dB} = \frac{\pi}{2M}$. This condition will reduce the amplitude distortion around the transition frequencies $\frac{(k+1)\pi}{M}$, where $k = 0, 1, \ldots, M-1$ [1].

Cosine modulated non-uniform filter banks

The non-uniform filter banks decompose the input signal into subbands of unequal bandwidths. The structure of an $\widetilde{M}$ channel cosine modulated non-uniform filter bank is shown in Fig. 1. A set of M analysis filters $\widetilde{H}_k(z),\ 0 \leqslant k \leqslant \widetilde{M}-1$ decomposes the input signal into $\widetilde{M}$ subbands. A set of synthesis filters $\widetilde{F}_k(z),\ 0 \leqslant k \leqslant \widetilde{M}-1$ combines the $\widetilde{M}$ subband signals. The decimation ratios are not equal in all the subbands. The $\widetilde{M}$ channel non-uniform design is obtained from the M-channel uniform CMFB by merging appropriate channels [10]. For maximally decimated filter banks, the decimation factors should satisfy the condition $\sum_{k=0}^{\widetilde{M}-1} \frac{1}{M_k} = 1$.

The non-uniform bands are obtained by merging the adjacent analysis and synthesis filters. Consider the analysis filter $\widetilde{H}_i(z)$, which are obtained by merging l_i adjacent analysis filters.

$$\widetilde{H}_i(z) = \sum_{k=n_i}^{n_i+l_i-1} H_k(z), \quad i = 0, 1, \ldots, \widetilde{M}-1 \tag{10}$$

Here, n_i is the upper band edge frequency ($n_0 = 0 < n_1 < n_2 < \cdots < n_{\widetilde{M}} = M$) and l_i is the number of adjacent channels to be combined. The synthesis filter $\widetilde{F}_i(z)$, is obtained in a similar way.

$$\widetilde{F}_i(z) = \frac{1}{l_i} \sum_{k=n_i}^{n_i+l_i-1} F_k(z), \quad i = 0, 1, \ldots, \widetilde{M}-1 \tag{11}$$

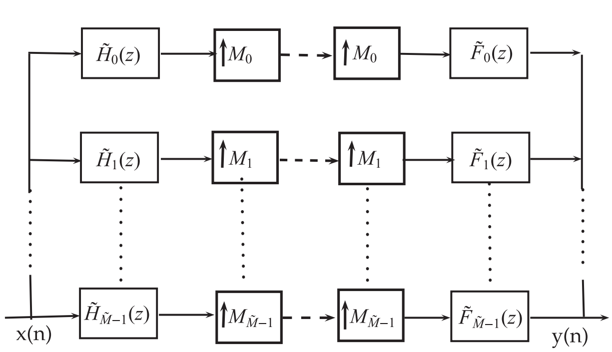

Fig. 1 Cosine modulated non-uniform filter bank.

The corresponding decimation factor M_i, is given by $M_i = \frac{M}{l_i}$. The condition to be satisfied for alias cancelation is that l_i and n_i are chosen such that n_i is an integral multiple of l_i, for all $i = 0, 1, \ldots, \widetilde{M}-1$ [10].

In uniform CMFB, the spectrums of the aliased components of the analysis filters do not have passband overlapping with the spectrums of synthesis filters. For non-uniform filter banks the overlapping occur in an irregular pattern. Hence constraints are imposed on l_i, to eliminate the undesired passband overlaps of the analysis filters. The passbands of $\widetilde{H}_i(zW^{2l_i l}),\ l = 1, 2, \ldots, M_i - 1$ and $\widetilde{F}_i(z)$ do not overlap if and only if n_i is an integral multiple of l_i for $i = 0, 1, \ldots, \widetilde{M}-1$.

Design of prototype filter

The popular techniques available for the design of linear phase FIR filters are the window method and optimum approximation methods. The optimum approximation methods can be classified as weighted Chebyshev approximation or minimax method and weighted least square approximation. Window method is a straight forward technique that involves a closed form expression, whereas minimax and least square approaches minimize the error function in an iterative manner to obtain the optimal filter.

The prototype filter design using weighted Chebyshev approximation using a linear search technique is proposed in [22]. The prototype filter for cosine modulated filter bank using different types of windows and with different objective functions in an iterative manner was previously recorded [23,24]. The prototype filter design using WCLS approximation is proposed in [25]. In this paper, the prototype filter is designed using Weighted Chebyshev approximation, Kaiser window approach and weighted constrained least square technique, for the same specifications. The passband and stopband edge frequencies are iteratively adjusted, with fixed transition width to satisfy the 3-dB condition [24]. To eliminate the amplitude distortion, the condition to be satisfied by the prototype filter, $P_0(z)$ is given below

$$|P_0(e^{j\omega})|^2 + |P_0(e^{j(\omega - \frac{\pi}{M})})|^2 = 1, \quad \text{for } 0 \leqslant \omega \leqslant \frac{\pi}{M} \tag{12}$$

From the above relation it can be shown that

$$|P_0(e^{\frac{j\pi}{2M}})| \approx 0.707 \tag{13}$$

The passband edge frequency [22], cut-off frequency [23] or both edge frequencies simultaneously with fixed transition width, can be iteratively adjusted with small step size to satisfy the condition (13) within a given tolerance value.

Design example

Design specifications

Number of channels: 8.
Roll-off: 0.809.
Stopband attenuation: 60 dB.
Passband ripple: 8.6×10^{-3} dB.

Initially an 8 channel uniform CMFB is designed, in which the prototype filter is designed using window method, weighted

Chebyshev approximation and WCLS approximation. Four channel and five channel non-uniform filter banks with decimation factors (8, 8, 4, 2) and (4, 4, 8, 8, 4) respectively are designed by appropriately merging the filters of 8 channel CMFB. The different other non-uniform combinations that can be obtained from an 8-channel uniform CMFB are with decimation factors (2, 4, 8, 8), (8, 8, 4, 2), (4, 4, 2), (2, 4, 4), (8, 8, 4, 4, 4), (4, 4, 4, 8, 8) and (8, 8, 4, 4, 8, 8).

Window approach

This is a simple method to design FIR filter, with minimum amount of computational effort. The filter design using window method in which the ideal impulse response is multiplied by the window function is given by

$$p_0(n) = h_{id}(n)w(n), \quad 0 \leqslant n \leqslant N \tag{14}$$

$p_0(n)$ are the required filter coefficients. $h_{id}(n)$ is the impulse response of the ideal filter with cut-off frequency ω_c and $w(n)$ is the window function with length N.

$$h_{id}(n) = \frac{\omega_c}{\pi}\left(\frac{\sin(\omega_c n)}{\omega_c n)}\right), \quad \infty \leqslant n \leqslant \infty \tag{15}$$

Different window functions (Kaiser, Blackman, etc.) are available for limiting the infinite length impulse response of the ideal filter. In this paper, the prototype filter designed with the window method is by using the Kaiser window. The window function $w(n)$ is given by

$$w(n) = \frac{I_0(\beta)\sqrt{1-((n-0.5N)/0.5N)^2)}}{I_0(\beta)}, \quad 0 \leqslant n \leqslant N \tag{16}$$

where $I_0(\cdot)$ is the zeroth order modified Bessel function. Window method sometimes results in more number of coefficients.

The responses of the analysis filters and the amplitude distortion plot for the 4 channel CMFB (8, 8, 4, 2) using Kaiser window for the design of the prototype filter, are shown in Figs. 2 and 3 respectively. The responses of the analysis filters and the amplitude distortion plot for the 5 channel CMFB (4, 4, 8, 8, 4) using Kaiser window for the design of the prototype filter, are shown in Figs. 4 and 5 respectively.

Weighted Chebyshev approximation

The linear phase FIR filter design problem can be formulated as a Chebyshev approximation which minimizes the maximum error over a set of frequencies. A set of coefficients is determined such that the maximum absolute value of the error is minimized over the frequency bands in which the approximations is performed.

Parks McClellan algorithm is the linear phase FIR filter design algorithm developed by McClellan et al. [26] using weighted Chebyshev approximation. It is an iterative algorithm for finding the optimal Chebyshev FIR filter. The algorithm designs equiripple FIR filter which minimizes the maximum error between the ideal and actual filters. The ripples are evenly distributed over the passband and stopband. The computational effort is linearly proportional to the length of the filter.

The responses of the analysis filters and the amplitude distortion plot for the 4 channel CMFB (8, 8, 4, 2) using weighted Chebyshev approximation for the design of the prototype filter, are shown in Figs. 2 and 3 respectively. The responses of the analysis filters and the amplitude distortion plot for the 5 channel CMFB (4, 4, 8, 8, 4) using weighted Chebyshev approximation for the design of the prototype filter, are shown in Figs. 4 and 5 respectively.

Weighted Constrained Least Square (WCLS) Technique

The weighted least square (WLS) design minimizes the energy in the ripples in both the passband and stopband. The WCLS is the extended version of the WLS design approximation. The WCLS is a technique proposed by Selesnick et al. [27] for the design of a linear phase filter. This method is also an iterative algorithm. In each iteration a modified design is performed using Lagrange multipliers and the constraints are checked. It also includes the verification of Kuhn–Tucker conditions, so that all the multipliers are non negative. FIR filters can be designed with relative weighting of the error minimization in each band. An important performance controlling parameter is the error ratio κ given by

$$\kappa = \frac{\int_0^{\omega_p} |P_0(e^{j\omega}) - 1|^2 d\omega}{\int_{\omega_s}^{\pi} |P_0(e^{j\omega})|^2 d\omega} \tag{17}$$

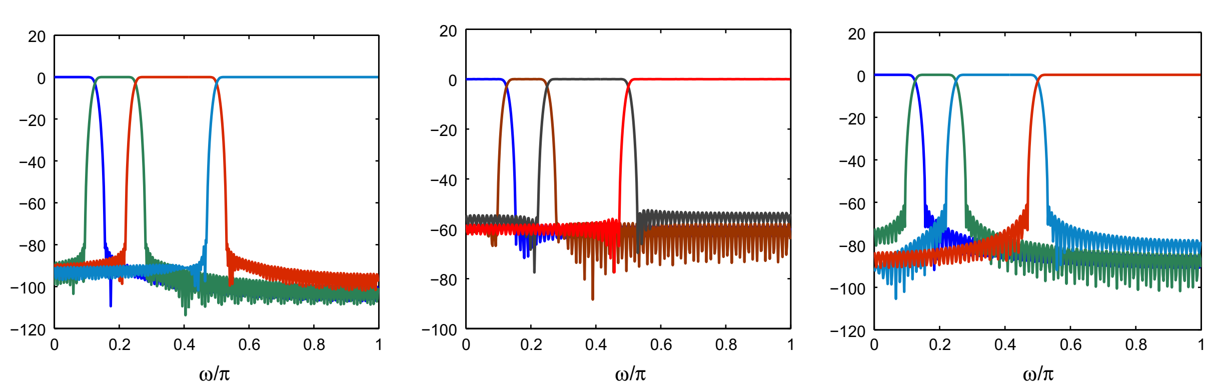

Fig. 2 Frequency response of analysis filters (8, 8, 4, 2) (Window, Chebyshev and WCLS).

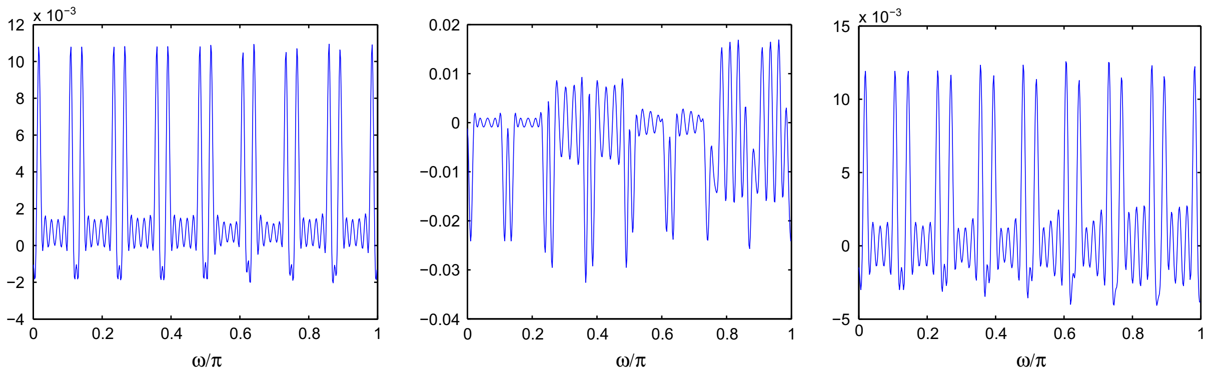

Fig. 3 Amplitude distortion function plots (8, 8, 4, 2) (Window, Chebyshev and WCLS).

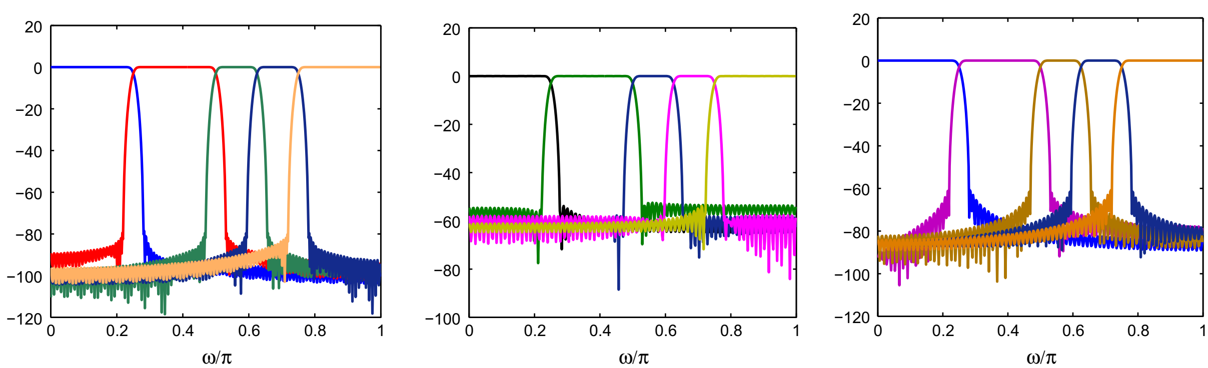

Fig. 4 Frequency response of analysis filters (4, 4, 8, 8, 4) (Window, Chebyshev and WCLS).

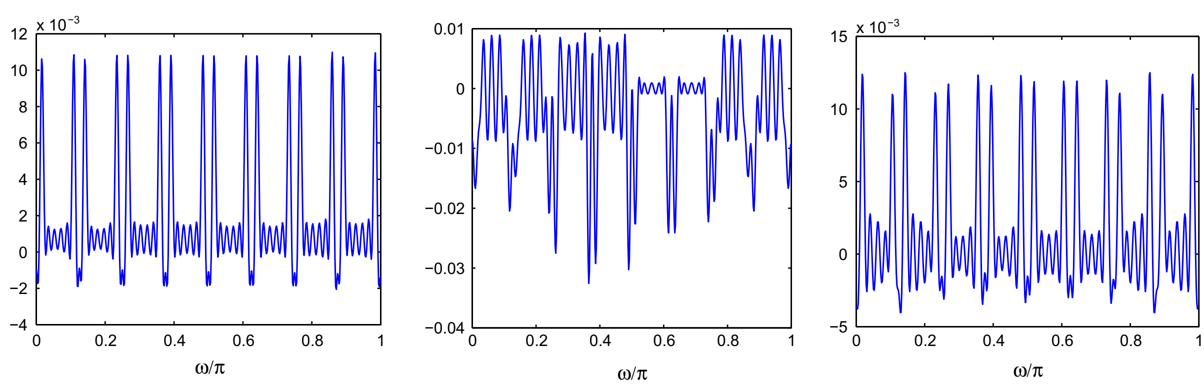

Fig. 5 Amplitude distortion function plots (4, 4, 8, 8, 4) (Window, Chebyshev and WCLS).

For small values of κ, the passband L_2 error is reduced whereas the stopband error is increased. In the case of large values of κ, the passband L_2 error is increased whereas the stopband error is reduced.

The responses of the analysis filters and the amplitude distortion plot for the 4 channel CMFB (8, 8, 4, 2) using WCLS approximation for the design of the prototype filter, are shown in Figs. 2 and 3 respectively. The responses of the analysis filters and the amplitude distortion plot for the 5 channel CMFB (4, 4, 8, 8, 4) using WCLS approximation for the design of the prototype filter, are shown in Figs. 4 and 5 respectively. The performance comparison of proposed prototype filters for four channel non-uniform CMFB (8, 8, 4, 2) with existing design method using Kaiser window is given in Table 1.

Multiplier-less design of non-uniform CMFB

If the coefficients in the filters are represented using SPT terms, the multipliers can be implemented using shifters and adders [28]. CSD contains minimum number of SPT terms and results in reduced number of shifters and adders [29]. For any decimal number, the corresponding CSD representation has a unique

Table 1 Performance comparison of the proposed prototype filters using continuous coefficients (8,8,4,2) with existing method.

	Weighted Chebyshev (**Proposed**)	WCLS approach ($\kappa = 0.1$) (**Proposed**)	Window method [38]
SB attn.(dB)	60.65	61.49	79.65
PB ripple (dB)	9.2×10^{-4}	4.2×10^{-3}	1.6×10^{-3}
Err. in amp. dist.[a] (8,8,4,2)	3.9×10^{-3}	2.8×10^{-3}	2.5×10^{-3}
Err. in amp. dist. (4,4,8,8,4)	4.2×10^{-3}	2.9×10^{-3}	2.5×10^{-3}
Filter order	154	154	198

[a] Error in amplitude distortion.

SPT representation. CSD is a radix-2 representation within the digit set $\{1, 0, -1\}$. CSD has a canonical property that the non-zero digits (1 and -1) will be never adjacent. The number of non-zero digits will be minimum. As a result, minimum number of adders and shifters are required for the implementation. The coefficients of all the prototype filters are converted to finite word length CSD representation with restricted number of SPT terms.

Look-up-table approach

A look-up-table approach is used for the fast conversion of the filter coefficients to their corresponding CSD equivalent with restricted number of non-zero terms [30]. A typical look-up-table entry for 16 bit CSD conversion is shown in Table 3. The look-up-table consists of four fields: an index, CSD equivalent, corresponding decimal and number of non-zeros present in the CSD equivalent. The coefficients can be converted to their nearest values in the CSD space with specified number of non-zero terms, using the look-up- table.

Performance comparison

The filter coefficients are converted to finite precision CSD using look-up-table [30]. The performance of CMFB using Kaiser window for different word lengths are given in Table 2. The 12 bit CSD representation gives the worst performance with the lowest implementation complexity. The 16 bit CSD representation gives the best performance with the worst implementation complexity. Hence as a compromise between filter performance and implementation complexity, it is good to choose 14 bit CSD representation.

Objective function formulation

The optimization goal in the multiplier-less CMFB is to reduce the following objective functions.

$$F_1 = \max_{0<\omega<\frac{\pi}{M}} \left\{ |P_0(e^{j\omega})|^2 + |P_0(e^{j(\omega - \frac{\pi}{M})})|^2 - 1 \right\} \quad (18)$$

$$F_2 = \max_{\omega > \frac{\pi}{2M}} |P_0(e^{j\omega})| \quad (19)$$

$$F_3 = \max(0, n(x) - n_b) \quad (20)$$

$$\min \phi = \alpha_1 F_1 + \alpha_2 F_2 + \alpha_3 F_3 \quad (21)$$

The design problem is formulated as a multi objective unconstrained problem. The objective function given in (18) minimizes the overall amplitude distortion and (19) is to minimize the maximum error in the stopband of the filter and (20) is the constraint added to the objective function using the penalty method that reduces the number of SPT terms [31]. Here $n(x)$ denotes the average number of SPT terms in the filter coefficients and n_b is the required upper bound. Eq. (21) combines the three objective functions, where α_1, α_2 and α_3 are the trade-off parameters, which define the relative importance given to each term in the final objective function.

Optimization of non-uniform CMFB using modified meta-heuristic algorithms

The different modified meta heuristic algorithms used in this paper are Artificial Bee Colony (ABC) algorithm, Gravitational Search algorithm (GSA) and Harmony Search algorithm (HSA). The advantage of meta-heuristic algorithms is that the objective function need not be differentiable and continuous [32].

Optimization of non-uniform CMFB using modified ABC algorithm

ABC Algorithm is a population based search technique introduced by Karaboga and Basturk [33]. Employed Bees, OnLooker Bees and Scout Bees constitute the artificial colony of honey bees. Possible solution of the problem is represented as the food source and the corresponding fitness is the amount of the nectar of the food source. An employed bee is the bee

Table 2 Performance comparison of CSD coefficient CMFB for different wordlengths (weighted Chebyshev approach).

	12 bits CSD	14 bits CSD	16 bits CSD
Min. SB attn.	50.67	57.28	59.39
Max. PB ripple	1.6×10^{-2}	6.7×10^{-3}	1.6×10^{-3}
Max. amp. dist.	9.02×10^{-3}	3.9×10^{-3}	3.09×10^{-3}
Adders due to SPT terms	67	116	171

Table 3 A typical look up table entry.

Index	CSD equivalent	Decimal equivalent	Number of non-zeros
8814	$2^{-1}+2^{-5}+2^{-7}-2^{-10}-2^{-12}+2^{-14}$ 100010100-10-101	0.5379	6

who goes to the previously visited food source. Employed bees choose a food source within the neighborhood of the food source in their memory. The new solution vector is formed adjacent to the existing vectors. Onlooker bee is the bee waiting in the dance area for taking the decision to choose a food source. Onlooker bees take the information provided by the employed bees regarding the fitness function. Onlooker bee selects the food source based on the fitness function. As a result, the food source with a high fitness value will get more onlookers. If the nectar quality of a food source is not improved after a certain number of iterations called the limit cycles, it is abandoned. The employed bee associated with the abandoned food source becomes a scout. The scout bee randomly finds a food source. The different phases involved in the optimization are given below [31].

Initialization

The prototype filter coefficients are CSD rounded and concatenated as a vector to form the initial food source. Only half the number of coefficients are used, since it is a linear phase filter. Initial random population is obtained by randomly perturbing this food source. The fitness value of each food source is evaluated and sorted according to its fitness value. N vectors with good fitness values are passed on to the next stage.

Employed bee phase

Employed bees choose a food source within the neighborhood of the food source in their memory. The new solution vector is formed adjacent to the existing vectors. The new food source at the ith position is obtained as follows:

$$v_{ij} = x_{ij} + \lfloor \phi \delta_{ij} \rfloor$$

where ϕ is the random variable within $[-1, 1]$ and δ_{ij} is defined as $\delta_{ij} = x_{ij} - x_{kj}$ x_{ij} is the jth parameter of the ith food source. The newly generated food sources are prevented from crossing the boundaries of the look up table [34]. If $v_{ij} < v_{lb}$ then $v_{ij} = v_{lb}$

If $v_{ij} > v_{ub}$ then $v_{ij} = v_{ub}$

where v_{lb} and v_{ub} are the lower and upper bounds of the look up table respectively. Now the fitness value of the new vector is evaluated and if it is better, then the old vector will be replaced by the new one. This is called greedy selection mechanism.

Onlooker bee phase

Onlooker bees take the information provided by the employed bees regarding the fitness function. Onlooker bee selects the food source based on the fitness function. The probability with which the onlooker bee chooses the food source was given by Manuel and Elias [34]

$$\frac{fit_i}{\sum_{j=1}^{N} fit_i} \tag{22}$$

where fit_i is the fitness function of the ith food source and N is the total number of food sources. As a result, the food source with high fitness value will get more onlookers. Like the employed bees, the onlooker bees also search for better food source in the neighborhood of the current food source. Similar to the employed bee phase, a greedy selection mechanism is done to select the new food source.

Scout bee phase

If the nectar quality of a food source is not improved after a certain number of iterations called the limit cycles, it is abandoned. The employed bee associated with the abandoned food source becomes a scout. The scout bee randomly finds a food source as given below.

$$v = \text{randi}([lb, ub], \text{'dim'})$$

where randi denotes the random integer values from the uniform discrete distribution within the interval [lb, ub] with the dimension of the food source specified by 'dim'.

Termination

Termination is achieved after a maximum number of iterations are reached, otherwise steps to are repeated. After the termination condition is satisfied, the food source with the best nectar quality is decoded using the look-up-table and the optimal filter coefficients are obtained.

Optimization of CMFB using modified HSA algorithm

Motivated by the music improvisation scheme, the Harmony Search algorithm (HSA) was developed by Z.W. Geem for the optimization of mathematical problems. By adjusting the pitches, the musician searches for a better state of memory. The decision variables are represented as musicians and solutions are represented as harmonics. Esthetics is equal to the fitness function and the pitch range denotes the range of values of the optimization variables.

A Harmony Memory (HM) is initialized, in which the solution variables resemble different musical notes. Musicians improve the harmonies for getting better esthetics. Similarly the Harmony Search algorithm explores the search space for finding the candidate solutions with good fitness value. In this algorithm a new solution is formed by the following three rules [35].

Table 4 Performance parameters of the non-uniform CMFB $(8, 8, 4, 2)$ using the Kaiser window.

	Max. PB ripple[a]	Min. SB attn[b]	Max. amp. dist.[c]	Run time (s)	Total	Multipliers adders [d]
Method in [38]	1.6×10^{-3}	79.65	2.5×10^{-3}		198	100
CSD rounded (3 SPTs)	4.5×10^{-3}	50.99	6.5×10^{-3}		299	0
Max. precision (7 SPTs)	4.8×10^{-3}	63.96	3.4×10^{-3}		323	0
Modified GA	4.7×10^{-3}	62.3	3.0×10^{-3}	63.67	315	0
Modified ABC	7.2×10^{-3}	63.3	2.94×10^{-3}	108.86	313	0
Modified GSA	6.8×10^{-3}	64.11	2.62×10^{-3}	406.56	313	0
Modified HSA	6.3×10^{-3}	62.42	3.11×10^{-3}	414.69	311	0

[a] Maximum passband ripple (dB).
[b] Minimum stopband attenuation (dB).
[c] Amplitude distortion.
[d] Hardware cost function.

1. *Memory consideration*: Selects any one value from the harmony memory.
2. *Pitch adjustment*: Selects an adjacent value from harmony memory.
3. *Random selection*: Selects a random value from the possible range.

The fitness function of the new harmony vector is evaluated and if it is found better, then the worst harmony vector is replaced with the new vector. Termination is reached either, when the stopband attenuation and error in amplitude distortion function reaches the limits specified or when a predetermined number of iterations are reached.

The various phases involved in HS algorithm are explained below [35].

Initialization

The Harmony Search algorithm is controlled using the parameters namely, Harmony Memory Size (HMS), Harmony Memory Considering Rate (HMCR) and Pitch Adjusting Rate (PAR). By perturbing the initial solution or initial harmony vector, various solutions are obtained. The initial number of harmony memory locations is taken to be an integer multiple of the number of memory locations (HMS). In this paper, a harmony vector in the harmony memory corresponds to the coefficients of the prototype filters of the FRM filter in the CSD encoded form. The fitness function of each vector is evaluated and the best solutions are passed on to the subsequent stages of optimization.

Harmony improvisation

A new harmony vector is generated from the harmony memory as follows

Memory consideration

Select the value of the ith element in the harmony vector in the harmony memory with a probability HMCR.

Pitch adjustment

Pitch adjustment is done with probability given in PAR as given below

$x_i^{new} = x_i + \lfloor rand(1, -1) FW(i) \rfloor$ $FW(i)$ is an arbitrary distance band width for the ith design variable and $rand(1, -1)$ is a uniformly distributed random number between -1 and 1.

Random selection

Generate random elements for the harmony vector with a probability $[1 - \mathrm{HMCR}]$.

Memory updates

The fitness function of the new harmony vector is evaluated and if it is found better, then the worst harmony vector is replaced with the new vector.

Termination

Termination is reached when the specified number of iterations are reached, otherwise steps ‘Harmony improvisation’ and ‘Memory updates’ are repeated.

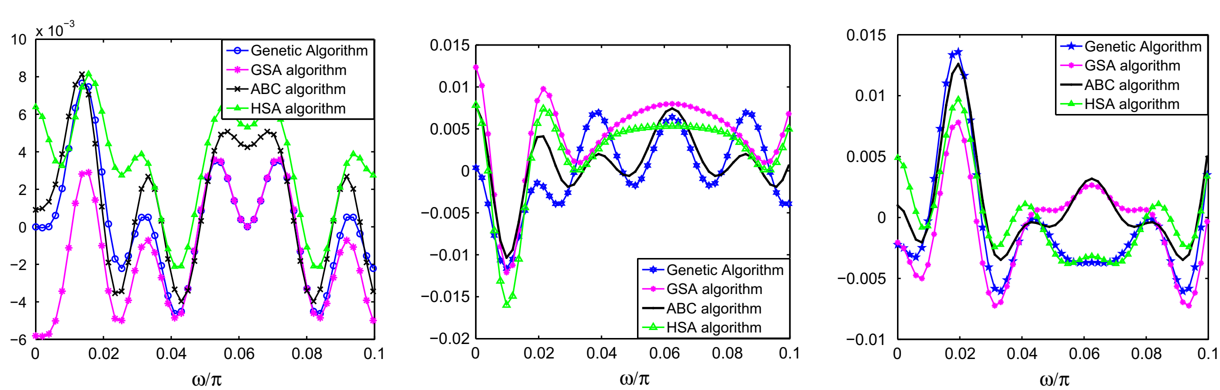

Fig. 6 Zoomed amplitude distortion plot of optimized non-uniform CMFB (8, 8, 4, 2) (Window, Chebyshev and WCLS).

Optimization of CMFB using modified GSA algorithm

GSA is a population based heuristic algorithm proposed by Rashedi in 2009 [36]. GSA is based on Newtonian law of gravity and motion [36]. A modified GSA algorithm for the design of 2D sharp wideband filter was previously proposed [37]. GSA can be considered as an artificial world of masses, where every mass represents a solution to the problem. A mass or agent is formed by the CSD encoded filter coefficients. Each mass has four specifications: position, inertial mass, active gravitational mass and passive gravitational mass. The position of mass is equivalent to the solution and the corresponding gravitational and inertial masses are determined by the fitness function. Masses attract each other by the force of gravity and the masses will be attracted by the heaviest mass which gives an optimum solution. The positions of the masses are updated in each iteration. Termination is reached either, when the stopband attenuation and error in amplitude distortion function reach the limits specified or when a predetermined number of iterations are reached.

Initialization

A mass or agent is formed by concatenating the CSD encoded coefficients of the prototype filter. Let N be the total number of agents or masses. Initial population is obtained by randomly perturbing the CSD encoded filter coefficients.

Fitness evaluation

The fitness of all the agents in each iteration is evaluated and the best and worst fitnesses are found at each iteration as follows.

$$worst(t) = \max_{j\epsilon 1,2,\ldots,N} fit_j(t) \tag{23}$$

$$best(t) = \min_{j\epsilon 1,2,\ldots,N} fit_j(t) \tag{24}$$

where $fit_j(t)$ represents the fitness value of the agent i at time t.

Compute the different parameters

The gravitational and inertial masses of each agent are calculated using the following equations

$$M_{ai} = M_{pi} = M_{ii} = M_i \tag{25}$$

$$i = 1, 2, \ldots, N$$

$$m_i(t) = \frac{fit_i(t) - worst(t)}{best(t) - worst(t)} \tag{26}$$

$$M_i(t) = \frac{m_i(t)}{\sum_{i=1}^{N} m_i(t)} \tag{27}$$

where M_{ai}, M_{pi} and M_{ii} represents the active gravitational mass, passive gravitational mass and inertial mass respectively of the ith agent.

Gravitational constant at each iteration t is computed by Eq. (28)

$$G(t) = G_0 e^{-\alpha t/T} \tag{28}$$

where T is the total number of iterations.

Calculate acceleration of agents

$F_{ij}^d(t)$ is the force acting on the mass 'i' from mass 'j' at time t in the dth dimension

$$F_{ij}^d(t) = G(t)\frac{M_{pi}(t)M_{ai}(t)}{R_{ij}(t) + \varepsilon}(x_i^d(t) - x_j^d(t)) \tag{29}$$

$R_{ij}(t)$ is the Euclidean distance between two agents i and j, ε is a small constant

The total force acting on an agent 'i' in a dimension of d is given as

$$F_i^d(t) = \sum_{j=1, j\neq i}^{N} rand_j F_{ij}^d(t) \tag{30}$$

$rand_j$ is a random number in the interval $[0, 1]$. The total force is expressed as a randomly weighted sum of the dth components of the forces exerted from other agents.

The acceleration of the ith agent at time t in the dth dimension is given by

$$a_i^d(t) = \frac{F_i^d(t)}{M_{ii}(t)} \tag{31}$$

where $M_{ii}(t)$ is the inertial mass

Update the velocity and position of agents

The velocity of the agent in the next iteration is represented as a fraction of its current velocity added to its acceleration. The new position and velocity are calculated as

$$v_i^d(t+1) = rand_i \times v_i^d(t) + a_i^d(t) \tag{32}$$

$$x_i^d(t+1) = \lfloor x_i^d(t) + v_i^d(t+1) \rfloor \tag{33}$$

Table 5 Performance parameters of the CMFB $(8, 8, 4, 2)$ using the weighted Chebyshev approximation.

	Max. PB ripple[a]	Min. SB attn[b]	Max. amp. dist.[c]	Run time (s)	Total adders [d]
Continuous coefficients	9.2×10^{-4}	60.65	3.9×10^{-3}		154
CSD rounded (3 SPTs)	2.1×10^{-2}	45.96	1.07×10^{-2}		251
Max. precision (7 SPTs)	6.7×10^{-3}	57.28	3.09×10^{-3}		273
Modified GA	9.97×10^{-3}	56.25	5.8×10^{-3}	61.16	266
Modified ABC	6.96×10^{-3}	57.87	3.88×10^{-3}	88.18	266
Modified GSA	5.9×10^{-3}	57.86	4.62×10^{-3}	406.64	267
Modified HSA	4.35×10^{-3}	55.9	6.39×10^{-3}	451.37	268

[a] Maximum passband ripple (dB).
[b] Minimum stopband attenuation (dB).
[c] Amplitude distortion.
[d] Hardware cost function.

Table 6 Performance parameters of the non-uniform CMFB $(8,8,4,2)$ using the WCLS method.

	Max. PB ripple[a]	Min. SB attn[b]	Max. amp. dist.[c]	Run time (s)	Total adders[d]
Continuous coefficients	4.2×10^{-3}	61.49	2.8×10^{-3}		154
CSD rounded (3 SPTs)	2.2×10^{-2}	49.09	8.88×10^{-3}		250
Max. precision (7 SPTs)	3.8×10^{-3}	60.03	3.5×10^{-3}		275
Modified GA	6.07×10^{-3}	58.74	4.63×10^{-3}	55.96	265
Modified ABC	5.65×10^{-3}	61.3	3.5×10^{-3}	96.7	264
Modified GSA	8.86×10^{-3}	63.27	2.26×10^{-3}	401.8	263
Modified HSA	6.3×10^{-3}	58.82	4.43×10^{-3}	489.4	266

[a] Maximum passband ripple (dB).
[b] Minimum stopband attenuation (dB).
[c] Amplitude distortion.
[d] Hardware cost function.

The new positions are prevented from crossing the boundaries of the look up table.

If $v_{ij} < v_{lb}$, then $v_{ij} = v_{lb}$.

If $v_{ij} > v_{ub}$, then $v_{ij} = v_{ub}$

where v_{lb} and v_{ub} are the lower and upper bounds of the look-up-table respectively.

Termination

The program will be terminated when the maximum number of iterations is reached, otherwise steps to will be repeated.

Results and discussion

All the simulations are done using a Dual Core AMD Opteron processor operating at 2.17 GHz using MATLAB 7.12.0. The performances of all the three prototype filters after optimization in the CSD space are compared in terms of the worst aliasing distortion, error in amplitude distortion, stopband attenuation, passband ripple and also the implementation complexity in terms of adders. Since all the filters are linear phase filters, only half of the symmetrical coefficients are extracted and optimized. The optimization results are shown for the non-uniform combination of $(8,8,4,2)$.

Optimal performance of non-uniform CMFB using Kaiser window

The CSD rounded filter coefficients in finite word length is optimized for the combined objective function given in (21), using various modified meta heuristic algorithms. Table 4 compares the performances of the prototype filter in terms of minimum stopband attenuation and maximum passband ripple achieved and also compares the non-uniform CMFB $(8,8,4,2)$ for the maximum error in amplitude distortion and the run time attained. The zoomed amplitude function plot for all the algorithms are shown in Fig. 6. The implementation complexity is compared in terms of the total number of adders which is given in Table 4. From Table (4), it can be observed that GSA algorithm has got maximum stopband attenuation and least error in amplitude distortion and comparable passband ripple and complexity. But the runtime is more than that of ABC algorithm.

Optimal performance of CMFB using WCLS method

Table 6 compares the maximum passband ripple and minimum stopband attenuation obtained for the prototype filter design using WCLS method. The maximum error in amplitude distortion and runtime for the CMFB, optimized using various modified meta-heuristic algorithms are also shown. Table 6 gives the implementation complexity comparison in terms of total number of adders. Fig. 6 gives the zoomed amplitude distortion function plot for all the algorithms. It can be concluded that GSA algorithm gives good stopband attenuation and less error in amplitude distortion with a reasonable runtime. The performances and implementation complexity using ABC algorithm are also good and takes less run time for convergence.

Optimal performance of CMFB using weighted Chebyshev approximation

Table 5 shows the performance comparison of the CSD rounded prototype filter design using weighted Chebyshev approximation and optimized using different algorithms, in terms of passband ripple and stopband attenuation. The performance of CMFB in terms of maximum error in amplitude distortion function and run time are given. Fig. 6 is the zoomed amplitude distortion function plot. From Table 5, it is clear that both GSA and ABC algorithm are suitable for optimizing multiplier-less NPR non-uniform CMFB. GSA algorithm results in good performances with less implementation complexity, but at the cost of increased run time.

Conclusions

In this paper, totally multiplier-less NPR non-uniform cosine modulated filter banks are designed and optimized in the discrete space using various modified meta-heuristic algorithms. The prototype filters are designed using window method, weighted Chebyshev and weighted constrained least square technique. A comparative study of the non-uniform NPR CMFB in the finite precision space, using the different prototype filter design approaches and optimization using various modified meta-heuristic algorithms, has been done in this paper. The prototype filter designed using window method is found to have better performance characteristics, but at the expense of increased implementation complexity. The WCLS technique is found to have less implementation complexity in terms of adders compared to Kaiser window approach in the discrete space. The finite precision prototype filter designed using weighted Chebyshev approach has moderate performances and implementation complexity. All the three prototype filters are optimized in the discrete space using various modified meta-heuristic algorithms. Modified GSA algorithm is found to outperform all the other algorithms for the optimization of the multiplier-less non-uniform NPR CMFB.

Conflict of Interest

The authors have declared no conflict of interest.

Compliance with Ethics Requirements

This article does not contain any studies with human or animal subjects.

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